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Beyond Presenting Good Problems

How a Japanese Teacher Implements a Mathematics Task

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For several years teachers have been encouraged to use tasks that engage students in mathematical thought processes. For example, the recommendations by the National Council of Teachers of Mathematics (NCTM) in both the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and the *Principles and Standards for School Mathematics* (NCTM 2000) encourage teachers and curriculum developers to use problems that go beyond practicing routine procedures to problems that help students build mathematical connections and develop and apply mathematical concepts. The expectation is that, by incorporating problems that engage students in mathematical thought processes, teachers will provide students with opportunities to conjecture, reason, and develop new mathematical ideas. Engaging students in such discussions is relatively new for both teachers and students. To help teachers learn some useful strategies, this article offers examples and discussions of ways teachers can implement problems to help students engage in mathematical thought processes.

Tasks That Help Students Make Connections

The initial release of data from the Third International Mathematics and Science Video Study (TIMSS Video Study) promoted a lot of discussion

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about the teaching styles in Germany, Japan, and the United States (Stigler et al. 1999). After looking at the data and the sample lessons, many observed how U.S. instruction looked very procedural in nature, whereas Japanese instruction provided more opportunities for participating in the mathematical processes encouraged in the NCTM *Curriculum and Evaluation Standards* (1989) and later, the NCTM *Principles and Standards* (2000). One aspect that received a lot of attention was the types of problems students were asked to work on during the lessons. For example, the U.S. teachers appeared to be using problems that asked students to practice routine procedures, such as using formulas to calculate perimeter and area. The Japanese teachers, however, were using problems that asked students to apply and develop mathematical concepts. The tasks used in the Japanese lessons appeared to help students engage in classroom discussions that included making and testing conjectures and reasoning about important mathematical relationships.

Although using tasks designed to engage students in these mathematical thought processes is useful in developing an environment that helps students think about and discuss important mathematical ideas, research has shown that simply using these types of tasks will not promote these discussions without support (Stein, Grover, and Henningsen 1996; Stein and Lane 1996). In particular, Stein and her colleagues found that the thinking and reasoning implied by the task statement are not necessarily the thinking and reasoning students engage in while working on and talking about the task.

Reflecting on some of the differences in instruction from the TIMSS Video Study as well as the work by Stein and her colleagues, I became interested in understanding the kinds of mathematics tasks U.S. and Japanese teachers provided students as well as how these tasks were completed during the videotaped lessons. An analysis of a subset of lessons from the TIMSS Video Study, showed that although almost 40 percent of the tasks presented by U.S. teachers were designed to engage students in making connections, only about 5 percent of these tasks were implemented in a way that publicly discussed these mathematical connections. Instead, these tasks were typically completed by applying a given procedure (Smith 2000). Many of the Japanese teachers, however, were able to help students make connections by strategically encouraging students to use mathematically rich discussions: about 63 percent of the tasks Japanese teachers presented were designed to encourage students to make connections and 35 percent of these tasks were publicly completed in a way that promoted making those connections (Smith 2000). The large difference in how teachers implemented tasks in these two countries suggests that it would be helpful to look at how the Japanese teachers support tasks in a way that helps students make connections.

The next section includes two illustrations of lessons incorporating problems designed to engage students in rich mathematical discussions. One les-

son describes how a U. S. teacher tried to introduce an engaging and meaningful mathematics task. The other describes how a Japanese teacher used a similar problem in her lesson. The discussion looks at some specific aspects of each teacher's implementation to help teachers think about their own practice. The differences represent some important features for implementing problems that help students make mathematical connections. The reader should note that neither the U.S. nor Japanese illustrations occurred in any one lesson: rather, I have consolidated several of the TIMSS Video findings into the two scenarios in this article. Although the stories as presented are fiction, in each respective instance they are built from the TIMSS Video data representing U.S. and Japanese teaching.

Two Teachers' Lessons

Mrs. Jones's Problem-Solving Lesson

Mrs. Jones and Mrs. Peterson attended an exciting workshop last week, where they learned about choosing good problem-solving tasks for their students. In one of the books they reviewed, they found the prom dress task (fig. 8.1) and knew it was perfect. With the "big dance" only a month away, the context was ideal. Looking more closely at the mathematics, they thought that it tied in nicely with the units on writing equations and graphing lines they had recently covered and would help students make a transition into linear systems of inequalities. They were both certain their students would generate great mathematical ideas.

A few days ago, Veronica and Caroline were both asked to the prom. That night, they went out to shop for dresses. As they were flipping through the racks, they each found the perfect dress, which cost $80. When they showed each other their dresses, they realized they both wanted the same dress! Neither of them had enough that night, but each went home and devised a savings plan to buy the dress. Veronica put $20 aside *that night* and has been putting aside an additional $5 a day, since then. Caroline put aside $8 *the day after* they saw the dress and has put in the same amount every day since.

Today, their friend Heather asks each girl how much she has saved for the dress. She says, "Wow! Caroline has more money saved." How many days has it been since Veronica and Caroline began saving?

Fig. 8.1. Prom Dress task

After assigning the problem to the students, Mrs. Jones wanted to look at how the students were tackling the problem. As she walked around the class,

she noticed that most students were having trouble just getting started. To help them, she drew three columns on the chalkboard: the left column was labeled "Days Saving for Dress," the middle was labeled "Veronica," and the right was labeled "Caroline." She then asked,

Mrs. Jones: How much money has each girl saved on the first day?

Karen: Veronica has $25 because she had 20 to start with, and Caroline has $8 because she had nothing to start with.

Mrs. Jones: Okay, good. How about the second day?

Sean: 30 and 16.

Mrs. Jones: Can you explain that?

Sean: Veronica has $30, and Caroline has $16.

Mrs. Jones: Okay, now find how many days since they went shopping when Caroline has more money.

The students returned to work, and Mrs. Jones continued to walk around the room. As some students finished, she asked them to find another way to solve the problem. She later called the students together to discuss their solutions. One student filled in the table up to the seventh day and said, "On the seventh day Caroline has $56, but Veronica has only $55, so it was the seventh day." Mrs. Jones asked if there were any other solutions. No one offered one. She then asked if it had to be the seventh day or if it could be another day. Another student replied, "Well, it is the seventh day, because that is the day she has more money!" The class then began to stir, insisting it was the seventh day. Just then the bell rang.

After class, Mrs. Peterson and Mrs. Jones discussed how students worked on the problems. Mrs. Jones noted, "Class did not go very well. The only method the students used was to substitute numbers. I just couldn't get my students to think of any other methods. Also, they all think that the seventh day is the only day Caroline has more money, not just the first of many days when Caroline is ahead." Mrs. Peterson said she had an equally unsuccessful experience and commented, "We worked so hard in learning how to pick out good problems. How are we supposed to help students generate all the great mathematical ideas in the problem?"

The issue Mrs. Jones and Mrs. Peterson raise is all too familiar: There are many resources to help select problems designed to engage students in making mathematical connections — for example, Smith and Stein (1998) and Stein et al. (2000). However, the question most teachers face is how to use these mathematically rich and engaging tasks effectively. In the next section, I present an illustration of a Japanese teacher who used a similar task in a way that elicited important mathematical connections. As in the U.S. lesson, students are asked to generate mathematical ideas, building an understanding of linear systems with inequalities. In this Japanese lesson, however, the

teacher tries to use some strategies to help students formalize their mathematical ideas. Comparing the example above with the way some Japanese teachers help students develop mathematical ideas through problem solving can provide teachers with some tools they can use to help promote mathematical communication.

A Japanese Story: Mrs. Hamada's Class

After greeting the class, Mrs. Hamada asked the students to settle down and get their mathematics books out while she put the gumball task (fig. 8.2) on the board. When the class was ready, she had a student read the problem aloud and then drew two rectangles on the board. One rectangle was labeled "Ken," and the other was labeled "Brother." The students then counted out 18 pink circles and then recounted the pink circles by tens to represent that Ken was starting with 180 pieces of gum. The students then counted out 24 yellow circles and then recounted the circles by fives to show that his brother was starting with 120 pieces of gum. Mrs. Hamada then asked the students to think about the problem and try to find a solution.

> Ken and his brother enjoy chewing gum. One day, the boys go to the candy store and buy several packages of gum. Ken bought 18 ten-piece packages of gum, and his brother bought 24 five-piece packages of gum.
>
> Every day, each of the boys finishes one whole pack of gum. One day, they looked at how much gum each boy had. Ken noticed that his brother had more pieces of gum than he had. How many days has it been since the boys bought the gum?

Fig. 8.2. Gumball task

The students worked on this task while Mrs. Hamada went around the room to see what the students were doing. She noticed that many students had not described their solution methods very well and commented, "Be sure to explain your answer in a way that someone else could understand what you have done." About halfway through the class, Mrs. Hamada asked her students to present their solutions to the class.

Group 1: (*The students go up to the board and draw a third rectangle and moved one pink [10-piece package] and one yellow circle [5-piece package] from the two boys' boxes into the third rectangle, as in figure 8.3 – Group 1. Continuing this action, the students explained their work.*) We drew a box here to throw away the packages of gum they chewed. On the first day, Ken puts in one 10-piece package of gum and his brother puts in one 5-piece package of gum. At the end of the first

day, Ken has 170 pieces of gum and his brother has 115 pieces of gum. We kept doing this until his brother had more pieces of gum, then we counted the packages of gum (*the student points to pairings of circles that represented how many days had passed*)… 12 days.

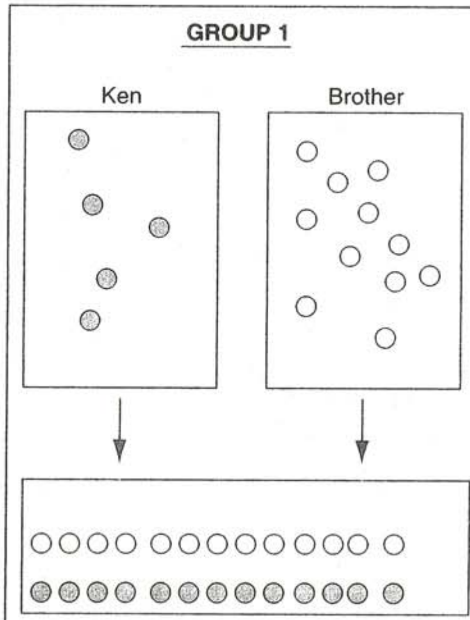
Mrs. Hamada: And how do you know the number of days from the number of circles?

Group 1: Because one circle is thrown out each day.

The teacher summed up their work, saying, "You took one circle from each boy, counting down by tens for Ken and by fives for his brother until his brother had more gum. This is good, but it could take a long time when the numbers get bigger. Did anyone find an easier way than this?"

Group 2: (*The students put the table in figure 8.3 – Group 2 on the chalk board.*) After the first day, Ken has 170 pieces of gum and his brother has 115. Then we kept doing this until the thirteenth day when his brother had more pieces of gum.

Mrs. Hamada: Okay, so you found out how many pieces of gum each boy had on the different days and showed it in this organized way. I think this would still take a long time if the numbers were bigger.



GROUP 2

Day	Ken	Brother
1	170	115
2	160	110
3	150	105
4	140	100
5	130	95
6	120	90
7	110	85
8	100	80
9	90	75
10	80	70
11	70	65
12	60	60
13	50	55

Fig. 8.3

It appeared as if Mrs. Hamada knew of another method she wanted her students to think about; she commented, “Now I wonder if any of you thought of a way to show how many pieces of gum each boy had every day. Many of you may not have thought of this way we will do, but that is okay, we will try it anyway. I would like you to add some columns to Group 2’s table like this (fig. 8.4—Before) and think of an equation Group 2 might have used to find out how many pieces of gum each boy had. What would day 1 look like?” One student from Group 2 responded, “We took 10 away from 180 for Ken and 5 away from 120 for his brother.” Mrs. Hamada filled this information in on the first line and asked the students to continue.

The students worked on this task at their seats. As Mrs. Hamada walked around the room, she noticed some students were confused. She said to the class, “Okay, now stop for a minute. What I want you to look at is the two numbers: How many pieces of gum Ken has and how many pieces of gum his brother has on each day and decide the computation you used to get that number. If you find one way to show this, think about if there are any ways that may be easier.” The students continued working, and the teacher had two students put their work on the board (fig. 8.4 – After). Once their work was up, Mrs. Hamada asked students to consider which equation could help them “if the number got really big.” The class decided that Student 2’s equation was most generalizable because “all you need to know is how many days so you can multiply it by how many pieces of gum are in each package, ten or five.” There wasn’t enough time to continue, so Mrs. Hamada asked the students to think of a more general way they could write how much gum each boy has on any given day.

DISCUSSION

There are many interesting ideas to talk about when comparing the Japanese teacher’s use of the gumball task with the U.S. teacher’s use of the prom dress task. The first is the similarity: Both teachers wanted students to use an interesting and meaningful problem to get their students to generate a mathematical idea. These teachers both provided problems in which students can generate mathematical concepts related to systems of linear inequalities. However, the teachers in these two classes differed in how they were able to help students work through the tasks. Four of the important differences follow.

- The Japanese teacher modeled the problem situation, whereas the U.S. teacher modeled a solution method.

The two teachers focused students’ attentions on different aspects of the problem. Mrs. Jones tried to help students get started by offering a possible solution method, but it appears to have focused their attention on just the one particular solution method, possibly contributing to their unwillingness

BEFORE

Day	Equation	Ken	Equation	Brother
1		170		115
2		160		110
3		150		105
4		140		100
5		130		95
6		120		90
7		110		85
8		100		80
9		90		75
10		80		70
11		70		65
12		60		60
13		50		55

AFTER

Day	Student 1	Student 2	Ken	Student 1	Student 2	Brother
1	180 – 10 = 170	180 – 10 = 170	170	120 – 5 = 115	120 – 5 = 115	115
2	170 – 10 = 160	180 – 20 = 160	160	115 – 5 = 110	120 – 10 = 110	110
3	160 – 10 = 150	180 – 30 = 150	150	110 – 5 = 105	120 – 15 = 105	105
4	150 – 10 = 140	180 – 40 = 140	140	105 – 5 = 100	120 – 20 = 100	100
5	140 – 10 = 130	180 – 50 = 130	130	100 – 5 = 95	120 – 25 = 95	95
6	130 – 10 = 120	180 – 60 = 120	120	95 – 5 = 90	120 – 30 = 90	90
7	120 – 10 = 110	180 – 70 = 110	110	90 – 5 = 85	120 – 35 = 85	85
8	110 – 10 = 100	180 – 80 = 100	100	85 – 5 = 80	120 – 40 = 80	80
9	100 – 10 = 90	180 – 90 = 90	90	80 – 5 = 75	120 – 45 = 75	75
10	90 – 10 = 80	180 – 100 = 80	80	75 – 5 = 70	120 – 50 = 70	70
11	80 – 10 = 70	180 – 110 = 70	70	70 – 5 = 65	120 – 55 = 65	65
12	70 – 10 = 60	180 – 120 = 60	60	65 – 5 = 60	120 – 60 = 60	60
13	60 – 10 = 50	180 – 130 = 50	50	60 – 5 = 55	120 – 65 = 55	55

Fig. 8.4

to provide additional solutions. Mrs. Hamada also helped her students get started: In her lesson, this meant having students model the problem by showing the number of packages of gum and relating that to the number of pieces of gum each boy had at the start. This enabled her to illustrate the situation before any actions have been taken rather than setting up the information in the problem in a way that suggested a specific method. Mrs. Hamada helped students ground their understanding in the problematic situation by helping them visualize the context and then asking them to find ways to resolve it.

- The Japanese students gave more detail in their explanations. When this was not spontaneous, the Japanese teacher specifically probed students to give more detailed and connected explanations.

Both Mrs. Jones and Mrs. Hamada asked students to explain at some point, but what counted as an explanation differed in these lessons. In Mrs. Jones's class, some mathematical connections go on in the background (likely in the students' heads) rather than being made public. Simply providing an answer was not acceptable in Mrs. Hamada's class: If students did not make connections, she asked questions that linked the pieces together. For example, when asking the students to give more detailed descriptions, she used phrases such as "to explain your answer in a way that someone else could understand what you have done." This type of prompting encouraged the students to look beyond getting an answer, to describing mathematical ideas. Moreover, this level of detail in explaining one's reasoning helped to fill in the gaps for a student who may have been struggling to understand the ideas being represented.

- The Japanese teacher helped students construct another solution method.

It appeared that Mrs. Hamada had another solution method she wanted students to explore, and she wanted to help her students figure it out. From a mathematical perspective, one can assume that Mrs. Hamada was trying to get students to consider using a common variable as a first step to comparing the expressions. In an effort to help students develop this idea, Mrs. Hamada tried to get students to begin thinking of the number of pieces of gum as variable expressions. Similar to when she initially introduced the problematic situation, Mrs. Hamada offered students a way to organize their information without directing students to use one particular method. (This could be seen when the two students constructed different ways to generate equations.) By helping students to see another method, she was offering even greater opportunities for students to consider different mathematical ideas. By constructing the method after they had grappled with some of the associated counting connected to it, the students were likely to develop an understanding of how and why this solution method works.

- In the Japanese lesson, the solution methods presented were analyzed and compared.

Mrs. Hamada and Mrs. Jones valued students' solution methods. Unfortunately, Mrs. Jones's students presented only one solution method, allowing little room for developing mathematical connections across solution methods. Because Mrs. Hamada's students presented more than one solution method, she was able to have them highlight mathematical relationships. For example, the similarity of the counting method used by Group 1 and the counting-down method in the table by Group 2 were stated. As another example, Mrs. Hamada asked students to build off the ideas presented by Group 2 to construct an equation. The idea of making connections ought to be at the heart of using these types of tasks. Prompting students to give more detailed explanations and discuss their reasoning allows all students opportunities to make sense of the task, the solution methods, and the mathematical relationships involved.

FINAL REMARKS

Teaching mathematics in a way that encourages students to make connections is a challenging endeavor. Moreover, the dynamic aspects of the classroom make it difficult to provide prescriptive lists of things to do to implement mathematical tasks designed to help students make connections. The illustrations presented here are meant to help teachers recognize ways of using tasks in the classroom that help make these mathematical connections explicit. The strategies presented can help teachers orchestrate mathematics tasks in a way that promotes discussions, moving students forward in their mathematical understandings. Several features to consider have been highlighted here: modeling the problem situation, not a solution method; prompting students for more detail in their explanations; helping students construct other solution methods; and analyzing and comparing solution methods. Readers are encouraged to use these suggestions, and those identified by others (e.g., Harris et al. 2001; Henningsen and Stein 1997; Smith and Stein 1998; Stigler and Hiebert 1999), as they work to incorporate mathematical tasks designed to help students make connections.

REFERENCES

Harris, Kimberly, Robin Marcus, Karen McLaren, and James Fey. "Curriculum Materials Supporting Problem-Based Teaching." *School Science and Mathematics* 101 (October 2001): 310–18.

Henningsen, Marjorie, and Mary Kay Stein. "Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathemat-